Cognitive Performance and Emotions*

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Abstract

An important literature is concerned with the relationship between cognition and emotions as determinants of human behavior. This paper contributes to this literature by studying the extent to which emotions are relevant for determining cognitive performance in a competitive environment. The analysis takes advantage of the empirical evidence obtained from a natural experiment which allows us to study whether seemingly small differences in emotional factors have an impact on cognitive performance. The setting is a chess match where two players play an even number of chess games against each other alternating the color of the pieces. Who starts the match with the white pieces is randomly decided. Therefore, in this randomized experiment there is no rational reason why average winning frequencies should be different from 50-50. Yet, observed frequencies are about 60-40 in favor of the player drawing the white pieces in the first game, and this effect is greater the more similar in cognitive skills the subjects are. The results suggest that systematic biases in cognitive decision making can persist in situations that are highly competitive, even those involving high-stake goods, sophisticated players and elaborate decision processes.

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1 Introduction

A fundamental question in the social sciences is the extent to which psychological elements are a part of human nature and, in particular, whether these elements are as important for understanding human behavior as the rationality considerations typically assumed in models that adhere to the rational man paradigm. Thus, one important challenge in the literature is to conclusively identify the relevance of psychological elements for explaining human behavior in real-life settings.[1] Nature does not often create the situations that allow a clear view of the psychological principles at work in human decisions. And when it does, naturally occurring phenomena are typically too complex to be empirically tractable in such a way that is possible to discern with sufficient clarity, and to measure with sufficient precision, the impact of emotional elements on human behavior.

This paper focuses on a specific type of human behavior: cognitive performance. Understanding cognition and the formation of cognitive skills is an important question in areas ranging from cognitive psychology and neuroscience to the literature on human capital, education and skill formation. For instance, as Kahneman (2011, pp. 20-21) notes, "psychologists have been intensely interested for several decades in the two modes of thinking [...] in the mind, System 1 and System 2. System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control. System 2 allocates attention to the effortful mental activities that demand it, including complex computations." From this perspective, this paper studies the relationship between these two systems at the limit of the distribution of sophistication in humans performing complex decision processes and finds that a systematic bias in cognitve decision making exists. As we will see, the analysis takes advantage of the opportunity provided by a randomized natural experiment to overcome a number of important obstacles that are often found in empirical research. To the best of our knowledge, this is the first study on cognitive performance in a competitive setting that takes

[1] See DellaVigna (2009) for an excellent survey, Rabin (1998), Camerer (2003), Camerer et al. (2003), Mullainathan and Thaler (2001) and other references therein. See Falk and Heckman (2009) for the important role of laboratory experiments.

advantage of a natural experiment. More precisely, the setting we study represents an unusually valuable opportunity for the following reasons:

First, it involves professional subjects who are characterized by a high degree of cognition at the specific task they perform as professionals (playing chess). Intuitively, their very high level of cognition at this task (comparable to the most advanced computers today) is relevant to study the role that psychological elements may play in humans even at the limit of the distribution of cognitive skills.

Second, the field setting concerns high stake decisions that subjects are familiar with, that really affect them, to which they are used, and that take place in their own real-life environment. In this sense, it involves a set of valuable characteristics in terms of stakes, familiarity and nature of the environment.

Third, the analysis concerns the study of a randomized natural experiment which is a powerful methodology in experimental sciences, including natural and medical sciences, that is not often available in the social sciences, especially in real-life environments. This ensures that the conditions for causal inference are satisfied since the treatment and control groups are determined via explicit randomization (Manski, 1995).

Fourth, from the perspective of observing and measuring behavior, the analysis uses a comprehensive dataset where choices, outcomes, and treatments can be measured precisely. Hence, the setting is not subject to any of the empirical difficulties that typically characterize real-life settings.

Fifth, measuring cognitive skills is typically difficult in the literature on cognition. An important advantage of the setting is that it is possible to measure with a high degree of precision the cognitive ability of the players at the task they perform. Players have a rating according to what is called the "ELO rating method," and this rating estimates quite precisely the probability that one player will outperform the other at the task.

Finally, the situation involves a tractable number of subjects (just two) competing at a game that is considered the ultimate intellectual sport (chess). The game they play has complete information and involves no chance elements. Subjects compete in the same setting and under identical circumstances and, as we will see next, the only difference is the randomly determined order in which they have to complete the task.

The rest of the paper is structured as follows. Section 2 describes the natural experiment. Section 3 describes the data. Section 4 presents the main empirical evidence. Section 5 is devoted to a formal discussion which contains a game-theoretical model of the task. Section 6 concludes.

2 The Natural Experiment

In a *chess match*, two players play an even number of *chess games*, typically about 8 to 10 games, against each other. Games are generally played one per day, with one or two rest days during the duration of the match. The basic procedure establishes that the two players *alternate* the colors of the pieces with which they play. In the first game, one player plays with the white pieces and the other with the black pieces. In the second game, the colors are reversed, and so on. Who plays with the white pieces in the first game is *randomly* determined, and this is the only procedural difference between the two players. According to the rules of FIDE (the Fédération Internationale d'Échecs, the world governing body of chess), the order is decided randomly under the supervision of a referee. This random draw of colors, which is typically conducted publicly during the opening ceremony of the match, requires that the player who wins the draw will play the first game with the white pieces. Therefore, the fact that players have no choice of order or color of the pieces makes it an ideal randomized experiment for establishing causality.

The explicit randomization mechanism used to determine which player begins with the white pieces in a sequence of games where both players have exactly the *same* opportunities to play the *same* number of games with the *same* colors, have the *same* stakes, are in the *same* setting and where all other circumstances are identical, suggests that we should expect both players to have, ceteris paribus, exactly the *same* probability of winning the match. That is, there is no rational reason why observed winning frequencies should be different from 50-50.[2] Yet, we find that this is not the case. As anticipation of the results, what we find instead is that winning probabilities are about 60-40 in favor of the player who plays with the white pieces in the first game, and hence in all the odd games of the match. As will be discussed, playing with the white pieces does give a slight advantage in a chess game and so, ceteris paribus, playing with the white pieces in the odd games means that that player is more likely to be leading during the course of the match.

With regard to the existing literature, this natural experiment takes advantage of the same design successfully used recently in the literature to study the behavior of other subject pools performing *non-cognitive* and/or *non-strategic* tasks.[3] The different nature of the tasks is, of course, what allows us to study the extent to which the randomly determined treatment (the order of colors) has an impact on *cognitive performance* in a *strategic* task.

3 Data

The dataset comes from *Chessbase*'s megabases, which are the most comprehensive databases in chess. They have detailed data on about 5 million games beginning in the XVIth century. We study *all* the matches during the period 1970-2010. We study this period since 1970 is the year when FIDE adopted the ELO rating system; that is, the year after which records on the cognitive ability of the players at this task, as measured by this rating, exist. The dataset is comprehensive as it includes *all* matches classified as such in Chessbase's megabases that perfectly fit the randomized experiment described earlier. That is, it does not include matches played versus a computer or matches without a perfect alternation of colors.[4]

[2] See Section 5 for a qualification of this statement.

[3] See Pope and Schweitzer (2011) and Apesteguia and Palacios-Huerta (2010) for evidence from golf and penalty kicks in soccer. See also Post, van den Assem, Baltussen, and Thaler (2008).

[4] The dataset does not include the final matches of World Championship cycles since it was generally the case that the incumbent, in case of a tie, remained the world champion, as well as two observations of matches with games played in the same day.

As indicated earlier, a valuable characteristic of the dataset is that it is possible to measure with a high degree of precision the cognitive ability or skills of the players at performing the task we study. Players have a rating according to the ELO rating method, and the difference between two players' ELO ratings is functionally related to an estimate of the probability that one of the players will beat the other should they play a chess game. Nowadays, the top 10 players in the world typically have an ELO rating between 2750 and 2825 points, the top 100 players a rating above 2650, and players with a rating above 2500 points are professionals who have the title of Grandmaster, which is the highest title that a player can achieve.

4 Empirical Evidence

We begin in the first subsection by studying all matches where players are professionals (have an ELO rating above 2500) and they are not exceedingly different in their skills. In a second subsection we present the results for the complete sample. In the final subsection we report additional evidence that relates the size of the effects to the similarity of the players in their skills.

4.1 Initial Evidence for Professionals

In order to have an initial sense of the evidence, we begin by studying all matches where players have an ELO rating above 2500 and the rating difference between the players is no greater than 100 points. Players with a rating above 2500 are professionals and the stakes in the matches they are involved in are high. Further, a match with an ELO difference above 100 points between the two players is a quite uneven match where the strong player is expected to with a very high probability regardless of other factors. This sample concerns 197 matches with a total of 1319 chess games.

As one would expect from the random treatment, the average quality of the players that begin with the white pieces (mean = 2620.1, std. deviation = 62.1) and the average quality of the players that begin with the black pieces (mean = 2616.7, std. deviation = 57.4) are statistically identical (p-value = 0.56).

Figure 1 shows the proportions of matches won by the player who begins playing the first game with the white pieces and by the player who begins with the black ones.

[Figure 1 here]

If the order had no effect on the outcome of a match, these proportions should be statistically identical. Yet, we find a significant and quantitatively important difference: the player who begins playing the first game with the white pieces wins 57.4 percent of the time. This is statistically different from 50 percent at the five percent significance level (p-value = 0.046). As we argue below, since this difference in performance or "average treatment effect" arises from the randomly determined difference in the order of play, it can be attributed exclusively to psychological effects resulting from the consequences of the playing order. Before discussing the potential mechanism that may generate these emotional effects we present additional empirical evidence.

In Figure 2 we split the data into "Elite" versus "Non-Elite" matches, and in matches for the World Championship versus other matches. "Elite" matches are those played by players with an ELO rating above 2600, and World Championship matches are matches belonging to the World Championship cycles organized by FIDE. These are two intuitive ways of selecting arguably more important matches, where the stakes are higher, and players are more skilled and have a deeper preparation.

[Figure 2 here]

We find that for Elite matches winning frequencies are 62-38 and for World Championship matches 67-33. These frequencies are statistically different from 50-50 at standard significance levels (for Elite matches p-value = 0.021, for World Championship matches p-value = 0.005). Thus, restricting to Elite and World Championship matches increases the magnitude and the significance of the effect.

4.2 Regression Results for the Complete Sample

This subsection first reports the complete set of results for the three different samples of professionals studied earlier: first for professionals with a rating above 2500, and second and third for the subsamples of Elite and World Championship matches. In each case we consider two different specifications, one with no restriction on the rating difference, and another with a maximum of a 100 ELO points difference. The most complete specification is, of course, the first column in the table.

[Table 1 here]

The results confirm the initial evidence presented in the previous figures: the effect of starting the match playing with the white pieces is positive and strongly significant in each of the regressions, typically with p-values < 0.01 and even p-values < 0.001. Further, the impact is greater in magnitude and statistically more significant in the more important matches (Elite and World Championship). As expected, the difference in ELO ratings between the players also has a positive and significant impact in the probability of winning a match in every regression specification.

In Table 2 we report the general results. In the first two columns we consider *all* the ELO-rated matches in Chessbase's megabase *regardless* of the ELO level of the players (with no upper bound on the ELO difference and with a maximum of a 100 point difference). These are the most general specifications. For the sake of potential interest, in the next four columns we study two minimum ELO levels (2200 and 2400).

[Table 2 here]

The results continue to confirm the strongly significant effect (p-values < 0.01) of starting the match playing with the white pieces. Similarly, the difference in ELO ratings continues to have a positive and significant impact on the probability of winning a match and the same is true for the two minimum ELO levels. Finally, we note that a small percentage of matches (not included in the sample) ended up tied. The same results are obtained in the corresponding ordered probit regressions with the three outcomes (win, loss, tie) when these matches are included. In the Appendix we report these results.

4.3 Additional Testable Implications

As indicated earlier, a valuable aspect of the natural experiment is that it is possible to measure with a high degree of precision the ability or skills of the players at performing the task. We take advantage of this opportunity and study the following testable prediction: given the undoubted role that other factors may play in determining the winner of a chess match, it should be the case that the psychological advantage of beginning with the white pieces significantly contributes to determining the outcome of a match *only* in relatively symmetric matches. That is, the more similar in strength two players are, the greater the effect should be. In other words, beginning with the white pieces should presumably tip the balance only when other factors are relatively similar, and in the limit the effect should be greatest when the two players are identical. We test this hypothesis. Figure 3 reports the evidence for the sample of matches studied earlier (subsection 4.1). Consistent with the hypothesis, we find that only in matches between players of similar ability are there significant differences in performance and, as predicted, the size of the effect steadily increases when players are more similar in cognitive skills.

[Figure 3 here]

These findings also contribute to explaining the results for Elite and World Championship matches presented earlier. These are matches where players have a deeper preparation and hence, conditional on their rating, their deeper preparation makes them more similar in skills during their matches.

5 Discussion: Effort and Strategic Risk Taking

In this section we discuss the role that non-psychological elements may play in the results. We first note that the randomly determined color of pieces generates just one, very specific type of asymmetry between the players. As will be discussed below, playing with white pieces confers a strategic advantage to win a chess game (in the sample, 30 percent of the chess games were won by the players with the white pieces and 17 percent by the player with the black pieces). Hence, what the random draw of colors means is that players who begin playing with the white pieces are randomly given a greater opportunity to lead in the match (and, conversely, those with the black pieces are given a greater opportunity to lag in the match). A chess match is a dynamic tournament and hence, in principle, it is possible that players may not have the same effort conditions during the match and/or may choose to take a different amount of risk depending on the score. In what follows we discuss the role that these non-psychological considerations may play in the results.

With respect to the idea that players can exert different effort during the match depending on the score, the design by FIDE of the typical chess match intends to ensure that all the games in a chess match are played under *identical* conditions and, in particular, that players have sufficient time to fully recover from the physical effort they exert: no more than one game is played each day and rest days are scattered during the duration of the match to ensure that players can play every single game in perfect physical conditions. Consistent with FIDE's objective, we know of no chess player who has ever argued that differences in physical effort may play a role in a match.

A more important consideration is the fact that players may adapt the risk they take during the match depending on the score. Understanding the role of strategic risk taking is not trivial and requires a formal model. Below we provide such a model. The model shows that, if anything, the role of risk *strengthens* the evidence obtained in the previous section on the impact of psychological elements. In other words, the effects documented in the previous section can be considered to be a *lower bound* on the actual impact of psychological elements on performance. The basic intuition for this result is the following. Lagging in the score may induce a player to take risks that he would otherwise not take in exchange for a greater probability to win a game and catch up in the score. Hence, the possibility of taking more risks and having more variable outcomes (e.g., increasing the chance of both winning *and* losing in exchange for a lower chance of tying) is an instrument at the disposal of the lagging player. This instrument, if anything, could help counterbalance any potential disadvantages given by the random determination of the colors. Clearly, the leading player can also tailor the risk he takes to the advantage that he has in the match and play more conservative strategies. However, no matter how conservative the leading player is, the lagging player can *always* drive the game into a win-lose lottery where the probability of winning is greater than if he had not chosen to take the additional risk.

This insight is already well-known in the literature in economics on the strategic choice of risk (variance and covariance) in dynamic competitive situations. See, for instance, Cabral (2002, 2003), Hvide (2002) and Hvide and Krinstiansen (2003). In what follows, we adapt the insights from this literature to the game of chess to show formally that in fact the possibility of choosing the risk that is taken during a match favors the lagging player. We present two models. The first one, which is a particular case of the second, shows the intuition for the result.

Consider a chess game between two identical players: white and black. Suppose that the players cannot adapt the risk they take (they can only take safe actions), and let W denote the probability that the player with the white pieces (white) wins and L be the probability that the player with the black pieces (black) wins. Hence, $1 - W - L$ is the probability that the match ends in a draw. As is well known, in chess it is strategically advantageous to play with the white pieces. This means that $W > L$. Empirically, this is strongly confirmed. For instance, in Chessbase's 4.8 million games megabases $W = 39\%$, $L = 31\%$, and for just the 165,000 games where both players have a ELO rating above 2500: $W = 28\%$ and $L = 18\%$.

Consider a chess match of T games, where T is an even number. In game 1, Player 1 plays with the white pieces and Player 2 with the black ones. In subsequent games the colors are alternated. Since a chess match is a constant-sum game, then, without loss of generality, we can assume that the utilities for each of the players are 1 if winning the match, 0 if losing, and 0.5 if they tie.[5]

[5] Since both players are identical, we can represent their preferences by the same utility functions. Moreover, without loss of generality, these utilities can be normalized so that the utility of a win is 1 and the utility of a loss is 0. Then, since we are in a constant-sum game, the utility of a tie has to be 0.5.

MODEL 1: The Simple Version

In a typical chess game, the first mover advantage gives the player with the white pieces *more* control over how "risky" the game will be. This is because he has more control over the type of "opening" that will be played.[6] In this first model we assume that he actually has *all* the control over the risk involved in the game. We relax this assumption in the general model later. We assume that the white player can increase his probability of winning by taking a risky action. This action increases his probability of winning by $R_w > 0$ and his probability of losing by αR_w, with $\alpha > 1$.[7]

First, note that Player 2 can guarantee for himself an expected payoff of at least 0.5 just by mimicking in the even games the choices made by Player 1 in the odd games. As we show below, it turns out that Player 2 can actually do strictly better than 0.5. Intuitively, this is because he can benefit from the fact that he has more information when he has to make his choices concerning risk.

Proposition 1. *Consider a match consisting of $T = 2$ chess games. Then, optimal play in this match leads to a higher expected payoff for Player 2 than for Player 1.*

Proof. We solve the game backwards. We start with Period 2, where Player 2 plays with the white pieces. If Player 2 lost the first game, he can only get a positive payoff by winning the second game, so he chooses the risky action. If the first game was a draw then, since the utility for a tie in the match is 0.5, Player 2 plays safe. That is, he does not want to transfer probability from the probability of drawing to the probability of winning if this entails transferring an even greater probability from the

[6] There is no much discussion about this assumption in the chess community. Yet, chess is too complex to provide a theoretical foundation for it. Nevertheless, it is relatively straightforward to document what expert players think about it. For instance, former world champion Vladimir Kramnik (June 2011, interviewed after the Candidates Matches to qualify to challenge the reigning world champion): "My white games were all pretty complicated, tense and full of fight. I am responsible for my white games, and I was always trying to find a way to fight with white, even if I did not get an advantage. But with black it is very difficult and incredibly risky to start avoiding drawish lines from the very beginning, because it can easily just cost you a point in a very stupid way [...] get a bad position, lose the game, lose the match and feel like an idiot? I didn't do it, but maybe at some point I should have. It is a difficult decision which can easily backfire at this level."

[7] Clearly, it must be the case that $W + L + (1 + \alpha)R_w \leq 1$.

probability of drawing to the probability of losing (recall that $\alpha > 1$). If Player 2 won the first game he just wants to minimize the probability of losing the second game, so he plays safe.

We look now at Period 1. We show that the expected payoff for Player 1 is less than 0.5 regardless of his action. The expected payoff for Player 1 if he chooses the safe action is:

$$W\big(1-(W+R_w)+\frac{W+R_w}{2}\big)+(1-W-L)(L+\frac{1-W-L}{2})+\frac{L^2}{2},$$

and this reduces to $\frac{1}{2}(1-WR_w)$, which is less than 0.5. His expected payoff when choosing the risky action is:

$$(W+R_w)(1-W-R_w+\frac{W+R_w}{2})+\big(1-W-L-(1+\alpha)R_w\big)(L+\frac{1-W-L}{2})+\frac{(L+\alpha R_w)L}{2},$$

and this reduces to $\frac{1}{2}\big(1-R_w^2-Rw(\alpha(1-W)-(1-W-L))\big)$, which is also less than 0.5 (recall that $\alpha > 1$). □

Corollary 1. *In a chess match consisting of T chess games, the expected payoff for Player 2 is greater than the expected payoff for Player 1.*

Proof. We already know that Player 2 can get an expected payoff of 0.5 simply by mimicking the actions of Player 1 in the previous game. Assume now that he always follows the mimicking strategy except in game T in the following case: If the score is tied at the end of game $T-2$, then he plays risky in game T if he lost game $T-1$ and plays safe otherwise. Proposition 1 implies that this is a profitable deviation from the mimicking strategy and, therefore, its expected payoff for player 2 is greater than 0.5. □

Note that in none of the above results we used the fact that $W > L$. Since Player 2 is the only one who can choose risk in Period 2, he is also the only one who can make an informed choice of risk. This "informational rent" is enough to give him an edge in the match. We show in the general model below that if *both* players have some control over risk in *both* periods, then the empirical fact that $W > L$ is crucial to prove that Player 2 has an advantage.

Model 2: The General Model

At a given chess game, both players can increase the probability of winning by taking a risky action. A risky action by white increases his probability of winning by $R_w > 0$ and his probability of losing by αR_w, with $\alpha > 1$. Similarly, a risky action by black increases his probability of winning by $R_b \geq 0$ and his probability of losing by αR_b.[8]

As noted earlier, in chess white has *at least as much* control over how "risky" a chess game is, that is $R_w \geq R_b$. We show below that this is a *sufficient* condition to obtain that Player 2 has an advantage in a chess match. Further, this condition is not necessary.

Note that since $W > L$ and $\alpha > 1$ the following two conditions are satisfied when $R_w \geq R_b$:

C1: $R_b < \frac{W}{L} R_w$.

C2: $R_b < R_w + \frac{R_w(\alpha-1)(1-W-R_b)}{L+R_w}$.

We show next that these two conditions suffice to give Player 2 an advantage in a two-game chess match. The intuition is that, although both players can control risk in both periods, the possibility of choosing a risky strategy is particularly valuable when a player is lagging in the score. Since $W > L$, Player 2 is more likely to be lagging in the score than Player 1 and, hence, he is the one more likely to benefit from risk taking in Period 2.

Proposition 2. *Consider a chess match with $T = 2$ games. When C1 and C2 are satisfied, optimal play leads to a higher expected payoff for Player 2. In particular, a sufficient condition for this result is that $R_w \geq R_b$.*[9]

Proof. We solve the game backwards. We start with Period 2.

[8] Clearly, it has to be the case that $W + R_w + \alpha R_b + L + R_b + \alpha R_w \leq 1$.

[9] Note that, when $R_b = 0$, conditions **C1** and **C2** are trivially satisfied and this result reduces to Proposition 1 in Model 1.

Suppose Player 1 won game 1: Then, in game 2, Player 1 is indifferent between a victory and a draw, so he plays safe. Player 2 is indifferent between a loss and a draw, so he plays risky. The expected utility for Player 1 in this case is:

$$u_1 = \frac{1}{2}(W + R_w) + 1(1 - (W + R_w))$$

Suppose Player 1 lost game 1: Following the above reasoning, Player 1 plays risky in game 2 and Player 2 plays safe. The expected utility for Player 1 in this case is:

$$u_2 = \frac{1}{2}(L + R_b)$$

Suppose that game 1 was a draw: Then, in game 2, since the utility for a tie in the match is 0.5, both players play safe. That is, they do not want to transfer probability from the probability of drawing to the probability of winning if it entails transferring an even greater probability from the probability of drawing to the probability of losing (recall that $\alpha > 1$). The expected utility for Player 1 in this case is:

$$u_3 = 1L + \frac{1}{2}(1 - (W + L)).$$

Period 1. Now we can compute the expected utilities for each combination of strategy profiles of the players in period 1. Let u_{SR} denote the expected utility of Player 1 when he plays safe and Player 2 plays risky and similarly the other utilities. Then:

$$\begin{aligned}
u_{SS} &= Wu_1 + Lu_2 + (1 - W - L)u_3 \\
u_{RR} &= (W + R_w + \alpha R_b)u_1 + (L + \alpha R_w + R_b)u_2 \\
&\quad +(1 - (W + L + (1 + \alpha)(R_w + R_b)))u_3 \\
u_{RS} &= (W + R_w)u_1 + (L + \alpha R_w)u_2 + (1 - ((W + R_w) + (L + \alpha R_w)))u_3 \\
u_{SR} &= (W + \alpha R_b)u_1 + (L + R_b)u_2 + (1 - ((W + \alpha R_b) + (L + R_b)))u_3
\end{aligned}$$

We now show that, given Period 2's optimal behavior, Player 2 can ensure for himself an expected utility higher than 0.5 by playing safe in period 1. Then, whatever Player 2's optimal choice is in period 1, it also gives him an expected payoff higher than 0.5. We compute the utilities of Player 1 with his two actions in period 1 when player 2 is playing safe. First, u_{SS} reduces to $\frac{1}{2}(1 + R_b L - R_w W)$, which is less than 0.5 when **C1** is satisfied. Second, u_{RS} reduces to

$$u_{RS} = \frac{1}{2}\big(1 + L(R_b - R_w) - R_w^2 + R_w(1 + (\alpha - 1)W + \alpha(-1 + R_b))\big),$$

which after some algebra can be rewritten as:

$$u_{RS} = \frac{1}{2}\big(1 - (L + R_w)(R_w - R_b) - R_w((\alpha - 1)(1 - W - R_b))\big),$$

which is less than 0.5 when **C2** is satisfied. □

Note that, although $R_w \geq R_b$ is a sufficient condition for the above results, it is clear from conditions **C1** and **C2** that Player 2 can also have an advantage *even* in cases where $R_w < R_b$. In other words, $R_w \geq R_b$ is not a necessary condition.

We conclude from these results that the observed differences in performance may be attributed exclusively to psychological effects resulting from the consequences of the playing order. Non-psychological effects operate in the opposite direction to the differences that have been documented.

6 Concluding Remarks

Understanding the various ways in which individual decision making deviates from standard notions of rationality is at the core of an important literature in the social sciences. And understanding cognition and the formation of cognitive skills is an important question in areas ranging from cognitive psychology and neuroscience to the literature on human capital, education and skill formation.

This paper takes advantage of the opportunity provided by a randomized natural experiment to contribute to these two areas of inquiry. Thus, it extends the analysis of an important literature into the area of cognitive performance. The results suggest that systematic biases in *cognitive* decision making can persist in and affect situations that are highly competitive, even those involving high-stake goods, sophisticated players and elaborate decision processes. Hence, the results suggest that in humans System 2, “the conscious, reasoning self that has beliefs, makes choices, and decides what to think about and what to do, [...] which believes itself to be where the action is,” remains subject to the impact of the“effortlessly originating impressions and feelings, impulses and associations, of System 1” (Kahneman, 2011) even at the limit of the distribution of sophistication and elaborate processes in the human mind.

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A Appendix: Ordered Probit Regressions

Tables A1 and A2 report the results of the ordered probit regressions when considering the matches that ended up tied. The set of outcomes is: Win, Loss, Tie.

[Tables A1 and A2 here]

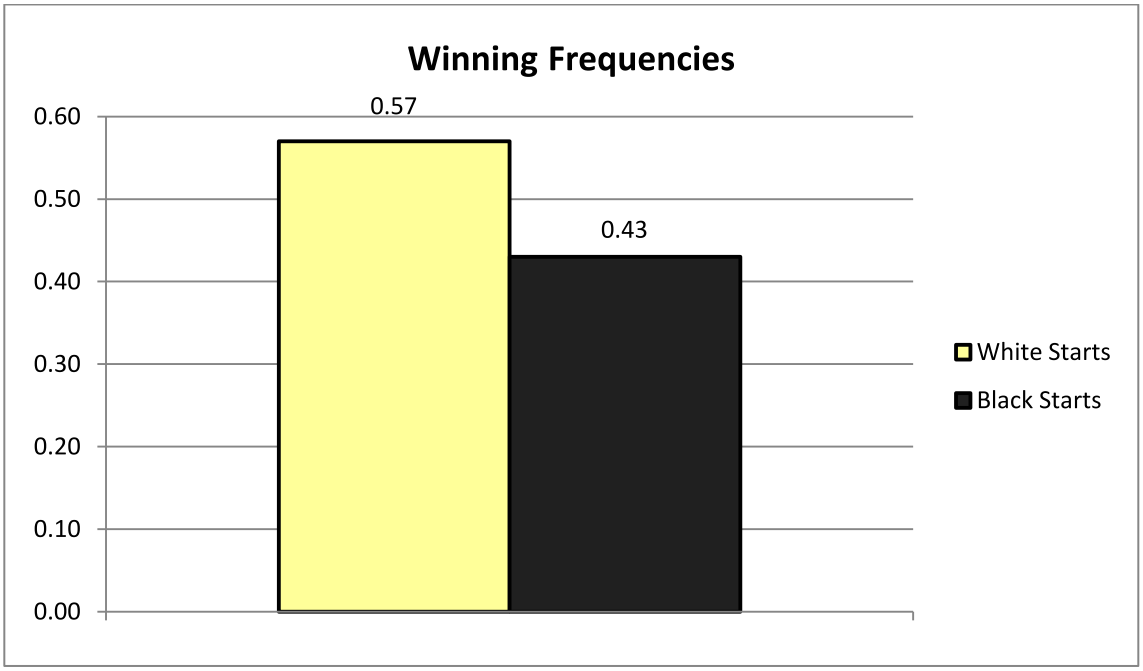

Fig. 1.

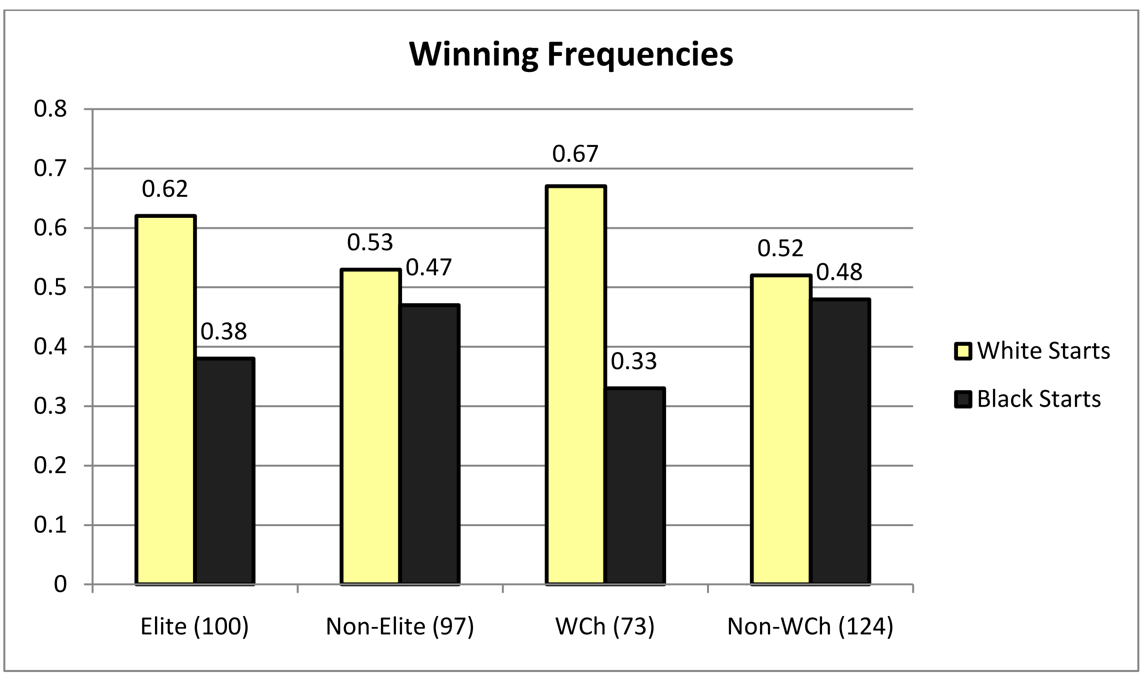

Fig. 2. The number of matches in each category are reported in parenthesis. The *p*-values of the proportions Chi-square tests are 0.02 (Elite matches), 0.68 (Non-Elite matches), 0.004 (World Championship matches) and 0.79 (Non-World Championship matches).

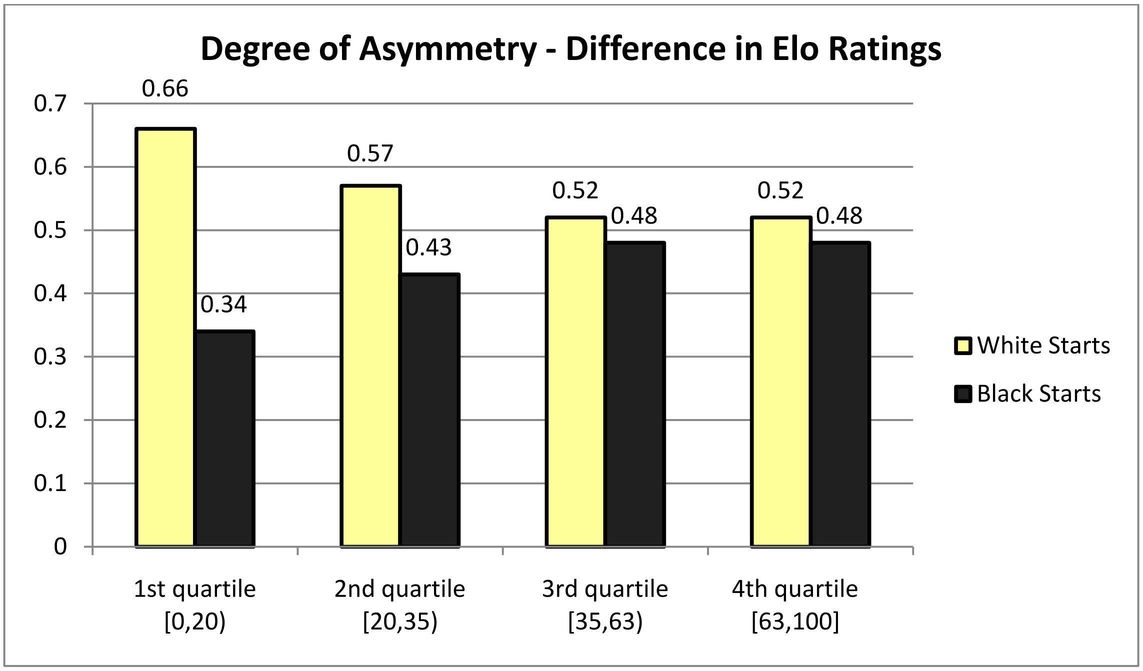

Fig. 3. Matches are sorted by the difference in ELO ratings between the two players, and then they are divided into four intervals (quartiles). The endpoint of the first interval is chosen so that it cuts off the lowest 25% of the sorted ELO differences. Similarly, the other endpoints cut off 50%, 75% and 100% respectively. The intervals are reported below each quartile. The number of matches in each quartile is 61, 42, 46 and 48 respectively. These differences in the number of matches across the intervals comes from the discreetness of the ELO ratings, which until the late 1999 moved in increments of 5 points. The *p*-values of the proportions Chi-square test are 0.02 (quartile 1), 0.44 (quartile 2), 0.88 (quartile 3) and 0.88 (quartile 4).

Table 1- Probit Regressions for Winners of a Chess Match

	Minimum ELO: 2500		**ELITE Matches Minimum ELO: 2600**		**World Championship Matches**	
Intercept	-0.120 (15.89)	-0.170 (16.68)	-0.284 (22.80)	-0.300 (23.10)	-0.376 (28.07)	-0.391 (29.27)
Starts with white pieces	**0.238**** **(0.129)**	**0.340***** **(0.134)**	**0.568***** **(0.190)**	**0.598***** **(0.192)**	**0.753***** **(0.224)**	**0.782***** **(0.226)**
Rounds	-0.000 (0.021)	0.000 (0.022)	-0.000 (0.030)	0.000 (0.031)	-0.000 (0.039)	-0.000 (0.040)
Year	-0.000 (0.0079)	0.000 (0.0083)	-0.000 (0.013)	0.000 (0.013)	-0.000 (0.016)	-0.000 (0.017)
ELO difference	**0.012***** **(0.001)**	**0.012***** **(0.001)**	**0.015***** **(0.0025)**	**0.013***** **(0.002)**	**0.012***** **(0.002)**	**0.011***** **(0.003)**
Player's ELO	0.000 (0.001)	0.000 (0.001)	0.000 (0.003)	-0.000 (0.003)	-0.000 (0.003)	-0.000 (0.003)
Maximum ELO Difference	None	100	None	100	None	100
N (matches)	234	197	116	100	79	73
Log-Likelihood	-252.15	-234.12	-116.25	-114.35	-83.19	-81.99
Akaike Information Criterion	516.29	480.24	244.51	240.69	178.38	175.97

Notes: *** denotes significant at the 1 percent significance level, ** at the 5 percent level, and * at the 10 percent level. The independent variables are the following: "Starts with white pieces" is a dummy variable that equals 1 if the player started playing the first game in the match with the white pieces; "Rounds" is the number of rounds in the match; "ELO difference" is the difference in ELO points with respect to the opponent at the time the match takes place; "Maximum ELO difference" is the maximum ELO difference between the players in the sample; "ELO points" are the ELO points of the player; and "Year" is the year the match takes place.

Table 2- Probit Regressions for Winners of a Chess Match

	Minimum ELO: None		**Minimum ELO: 2200**		**Minimum ELO: 2400**	
Intercept	-0.101 (11.50)	-0.130 (13.05)	-0.120 (11.96)	-0.128 (13.33)	-0.128 (14.01)	-0.162 (15.03)
Starts with white pieces	**0.202*** (0.094)**	**0.260*** (0.106)**	**0.240*** (0.098)**	**0.256** (0.108)**	**0.257** (0.112)**	**0.324*** (0.119)**
Rounds	-0.000 (0.015)	0.000 (0.017)	-0.000 (0.016)	-0.000 (0.017)	-0.000 (0.017)	-0.000 (0.019)
Year	-0.000 (0.005)	0.000 (0.006)	-0.000 (0.006)	-0.000 (0.006)	-0.000 (0.006)	-0.000 (0.007)
ELO difference	**0.008*** (0.000)**	**0.012*** (0.001)**	**0.009*** (0.000)**	**0.011*** (0.001)**	**0.011*** (0.001)**	**0.012*** (0.001)**
ELO points	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	-0.0000 (0.000)	-0.000 (0.000)
Maximum ELO Difference	None	100	None	100	None	100
N (matches)	438	314	414	303	315	252
Log-Likelihood	-465.62	-376.30	-434.89	-361.93	-335.55	-300.61
Akaike Information Criterion	943.24	764.61	881.79	735.86	683.11	613.23

Notes: *** denotes significant at the 1 percent significance level, ** at the 5 percent level, and * at the 10 percent level. The independent variables are the following: "Starts with white pieces" is a dummy variable that equals 1 if the player started playing the first game in the match with the white pieces; "Rounds" is the number of rounds in the match; "ELO difference" is the difference in ELO points with respect to the opponent at the time the match takes place; "Maximum ELO difference" is the maximum ELO difference between the players in the sample; "ELO points" are the ELO points of the player; and "Year" is the year the match takes place.

Table A1- Ordered Probit Regressions for Winners of a Chess Match

	Minimum ELO: 2500		**ELITE Matches Minimum ELO: 2600**		**World Championship Matches**	
Starts with white pieces	**0.250**** **(0.110)**	**0.338***** **(0.115)**	**0.603***** **(0.161)**	**0.618***** **(0.162)**	**0.703***** **(0.209)**	**0.740***** **(0.212)**
Rounds	-0.000 (0.016)	-0.000 (0.017)	-0.000 (0.022)	-0.000 (0.022)	-0.000 (0.031)	0.000 (0.031)
Year	-0.000 (0.001)	-0.000 (0.001)	-0.000 (0.002)	-0.000 (0.005)	-0.000 (0.004)	-0.000 (0.006)
ELO difference	**0.010***** **(0.001)**	**0.010***** **(0.001)**	**0.014***** **(0.002)**	**0.012***** **(0.002)**	**0.012***** **(0.002)**	**0.011***** **(0.003)**
ELO points	-0.000 (0.001)	0.000 (0.001)	-0.000 (0.002)	0.000 (0.004)	0.000 (0.003)	0.000 (0.004)
Loss/Tie	-0.069*** (0.000)	-0.026*** (0.000)	0.085*** (0.000)	0.082*** (0.000)	0.271*** (0.000)	0.115*** (0.001)
Tie/Win	0.317*** (0.044)	0.363*** (0.046)	0.511*** (0.068)	0.514*** (0.068)	0.430*** (0.001)	0.276*** (0.000)
Maximum ELO Difference	None	100	None	100	None	100
N (matches)	267	228	133	117	83	77
Log-Likelihood	-446.23	-413.88	-212.18	-208.66	-114.18	-123.91
Akaike Information Criterion	906.47	841.75	438.36	431.33	242.35	257.82

Notes: *** denotes significant at the 1 percent significance level, ** at the 5 percent level, and * at the 10 percent level. The independent variables are the following: "Starts with white pieces" is a dummy variable that equals 1 if the player started playing the first game in the match with the white pieces; "Rounds" is the number of rounds in the match; "ELO difference" is the difference in ELO points points with respect to the opponent at the time the match takes place; "Maximum ELO difference" is the maximum ELO difference between the players in the sample; "ELO points" are the ELO points of the player; and "Year" is the year the match takes place. "Loss/Tie" and "Tie/Win" are the corresponding threshold parameters of the ordered probit.

Table A2-Ordered Probit Regressions for Winners of a Chess Match

	Minimum ELO: None		**Minimum ELO: 2200**		**Minimum ELO: 2400**	
Starts with white pieces	**0.112* (0.077)**	**0.191** (0.089)**	**0.138* (0.080)**	**0.187** (0.091)**	**0.219** (0.093)**	**0.252*** (0.099)**
Rounds	0.000 (0.011)	0.000 (0.013)	-0.001 (0.012)	0.000 (0.013)	-0.000 (0.014)	-0.000 (0.015)
Year	0.000 (0.000)	0.000 (0.000)	-0.000 (0.000)	0.000 (0.000)	-0.000 (0.000)	-0.000 (0.000)
ELO difference	**0.006*** (0.000)**	**0.008*** (0.000)**	**0.006*** (0.000)**	**0.009*** (0.000)**	**0.008*** (0.000)**	**0.009*** (0.001)**
ELO points	0.000 (0.000)	-0.000 (0.000)	0.000 (0.000)	-0.000 (0.000)	-0.000 (0.000)	-0.000 (0.000)
Loss /Tie	-0.160*** (0.000)	-0.111*** (0.000)	-0.925*** (0.000)	-0.105*** (0.000)	-0.404*** (0.000)	-0.283*** (0.000)
Tie /Win	0.276*** (0.034)	0.308*** (0.037)	-0.495*** (0.034)	0.302*** (0.037)	0.012 (0.039)	0.136*** (0.041)
Maximum ELO Difference	None	100	None	100	None	100
N (matches)	511	369	481	354	364	296
Log-Likelihood	-895.74	-690.44	-837.48	-657	-625.47	-552.88
Akaike Information Criterion	1805.48	1394.88	1688.97	1328.23	1264.94	1119.77

Notes: *** denotes significant at the 1 percent significance level, ** at the 5 percent level, and * at the 10 percent level. The independent variables are the following: "Starts with white pieces" is a dummy variable that equals 1 if the player started playing the first game in the match with the white pieces; "Rounds" is the number of rounds in the match; "ELO difference" is the difference in ELO points with respect to the opponent at the time the match takes place; "Maximum ELO difference" is the maximum ELO difference between the players in the sample; "ELO points" are the ELO points of the player; and "Year" is the year the match takes place. "Loss/Tie" and "Tie/Win" are the corresponding threshold parameters of the ordered probit.